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Thermal Postbuckling Behavior of Anisotropic Laminated Cylindrical Shells with Temperature-Dependent Properties

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Thermal postbuckling analysis is presented for anisotropic laminated cylindrical thin shells of finite length. The temperature field considered is assumed to be a uniform distribution over the shell surface and through the shell thickness. Material properties are assumed to be temperature-dependent. The governing equations are based on the classical shell theory with a von-Kármán–Donnell-type of kinematic nonlinearity and including thermal effects. A singular perturbation technique is employed to determine buckling temperature and postbuckling load-deflection curves. The numerical illustrations concern the thermal postbuckling response of perfect and imperfect anisotropic laminated cylindrical shells with different values of shell parameters and stacking sequence. The joint effects played by anisotropy, nonlinear prebuckling deformations, and initial geometric imperfections are studied. The new finding is that there exists a compressive stress along with an associate shear stress when the anisotropic laminated cylindrical shell is subjected to heating, and all the results published previously need to be reexamined. The results reveal that the anisotropy has a significant effect on the buckling temperature and the postbuckling behavior of laminated cylindrical shells. The results also confirm that the thermal postbuckling equilibrium path is stable or weakly unstable and the shell structure is virtually imperfection-insensitive.

I. Introduction

COMPOSITE laminated shell structures are being increasingly used in aeronautical and aerospace construction. These shell structures are often subjected to severe thermal environments during launching and reentry and may have significant and unavoidable initial geometric imperfections. One of the problems is associated with the determination of the postbuckling behavior of laminated shells under such environmental conditions, which is essential for a better understanding and exploitation of their load-carrying capacity.

Many studies have been made on the modeling and analysis of composite laminated shells subjected to heating. Among those, Thangaratnam et al. [1] calculated the buckling temperature for composite laminated cylindrical and conical shells subjected to a uniform temperature rise by using semiloof finite elements. The influence of nonuniformly elevated temperature on the thermal buckling loads of antisymmetric laminated cylindrical shells was studied by Ma and Wilcox [2] using the Galerkin method. Birman and Bert [3] discussed the postbuckling response of stiffened and unstiffened composite plates and cylindrical shells under a compressive axial loads and a uniform temperature rise, but nonlinear prebuckling deformations and initial geometric imperfections were not included. Shen [4] presented a thermal postbuckling analysis for stiffened and unstiffened composite laminated cylindrical shells under three cases of thermal loading (i.e., uniform temperature rise and nonuniform parabolic temperature distributions varying in the circumferential and axial directions). In his study, both nonlinear prebuckling deformations and initial geometric imperfections were taken into account. This work was then extended to the case of laminated cylindrical shells with piezoelectric actuators [5] and to the case of functionally graded cylindrical shells with temperature-dependent properties [6]. Moreover, Ganesan and Kadoli [7] examined the thermal buckling and the influence of axisymmetric temperature on the nature frequency and active damping ratio of the piezolaminated cylindrical shells using the semi-analytical finite element method (FEM). In their study, the material properties are considered to be independent of temperature. However, the theories used in the preceding analyses are mostly extensions of the various isotropic models. Because the laminated composite cylindrical shells generally exhibit extension/twist and flexural/twist couplings when fiber angles exist that do not lie parallel to the cylindrical axis or in a circumferential plane [8–10], the traditional double Fourier expansion of the transverse displacement [2,11] [such as $\bar{W} = W_1 \sin(m\pi X/L)\sin(nY/R)$ or $W_1 \sin(m\pi X/L)\cos(nY/R)$, which is suitable for the cross-ply laminated cylindrical shells] is no longer a capable solution for asymmetric spiral buckling modes. Recently, Weaver et al. [9,10] suggested a solution formed as

$$\bar{W} = W_1 \sin(nY/R - knX/R) \tag{1}$$

where k is the slope of the spiral. It is worthy to note that Eq. (1) cannot satisfy boundary conditions such as simply supported or clamped at the end of the cylindrical shell and can be as approximate solutions [12].

It has been shown [13] that in shell thermal buckling as well as in shell compressive buckling, there exists a boundary-layer phenomenon in which prebuckling and buckling displacement vary rapidly. Recently, Shen [14–16] developed a boundary-layer theory for the buckling and postbuckling of anisotropic laminated thin shells under mechanical loading of axial compression and external pressure and torsion and found that there exists a compressive or circumferential stress along with an associate shear stress and twisting when the anisotropic laminated cylindrical shell is subjected to axial compression or lateral pressure. In contrast, there exists a shear stress along with an associate compressive stress when the anisotropic shell is subjected to torsion. Accordingly, we believe that there exists a compressive stress due to boundary constraints along with an associate shear stress when the shell is subjected to heating.

In the present study, the material of each layer of the shell is assumed to be linearly elastic, anisotropic, and fiber-reinforced, and the material properties are assumed to be linear functions of temperature. The governing equations are based on the classical shell theory with a von-Kármán–Donnell-type of kinematic nonlinearity and including the extension/twist, extension/flexural, and flexural/

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twist couplings. A singular perturbation technique is employed to determine the buckling temperature and thermal postbuckling load-deflection curves. The nonlinear prebuckling deformations and initial geometric imperfections of the shell are both taken into account. The numerical illustrations show the full nonlinear postbuckling response of anisotropic laminated cylindrical shells under uniform temperature rise.

II. Theoretical Development

A. Governing Equations

Consider a circular cylindrical shell with mean radius R, length L, and thickness t, which consists of N plies of any kind. The shell is referred to a coordinate system (X, Y, Z) in which X and Y are in the axial and circumferential directions of the shell and Z is in the direction of the inward normal to the middle surface. The corresponding displacement are designated by $\bar{U}$, $\bar{V}$, and $\bar{W}$. The origin of the coordinate system is located at the end of the shell on the middle plane. The shell is assumed to be relatively thin and geometrically imperfect and is subjected to a uniform temperature rise ΔT . Denoting the initial geometric imperfection by $\bar{W}^*(X, Y)$, let $\bar{W}(X, Y)$ be the additional deflection and $\bar{F}(X, Y)$ be the stress function for the stress resultants defined by $\bar{N}_x = \bar{F}_{,yy}$, $\bar{N}_y = \bar{F}_{,xx}$, and $\bar{N}_{xy} = -\bar{F}_{,xy}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

Based on classical shell theory (i.e., transverse shear deformation effects are neglected) with a von Kármán type of kinematic nonlinearity and including thermal effects, the nonlinear differential equations for an anisotropic cylindrical shell can be expressed in terms of a stress function $\bar{F}$ and transverse displacement $\bar{W}$, along with initial geometric imperfection $\bar{W}^*$. These equations in the Donnell sense are

$$\tilde{L}_{11}(\bar{W}) + \tilde{L}_{12}(\bar{F}) - \tilde{L}_{13}(\bar{N}^T) - \tilde{L}_{14}(\bar{M}^T) - \frac{1}{R}\bar{F}_{,xx} = \tilde{L}(\bar{W} + \bar{W}^*, \bar{F}) \tag{2}$$

$$\tilde{L}_{21}(\bar{F}) - \tilde{L}_{22}(\bar{W}) - \tilde{L}_{23}(\bar{N}^T) + \frac{1}{R}\bar{W}_{,xx} = -\frac{1}{2}\tilde{L}(\bar{W} + 2\bar{W}^*, \bar{W}) \tag{3}$$

where

$$\begin{aligned}
\tilde{L}_{11}() &= D^*_{11}\frac{\partial^4}{\partial X^4} + 4D^*_{16}\frac{\partial^4}{\partial X^3\partial Y} + 2(D^*_{12} + 2D^*_{66})\frac{\partial^4}{\partial X^2\partial Y^2} + 4D^*_{26}\frac{\partial^4}{\partial X\partial Y^3} + D^*_{22}\frac{\partial^4}{\partial Y^4}\\
\tilde{L}_{12}() &= \tilde{L}_{22}() = B^*_{21}\frac{\partial^4}{\partial X^4} + (2B^*_{26} - B^*_{61})\frac{\partial^4}{\partial X^3\partial Y} + (B^*_{11} + B^*_{22} - 2B^*_{66})\frac{\partial^4}{\partial X^2\partial Y^2} + (2B^*_{16} - B^*_{62})\frac{\partial^4}{\partial X\partial Y^3} + B^*_{12}\frac{\partial^4}{\partial Y^4}\\
\tilde{L}_{13}(\bar{N}^T) &= \frac{\partial^2}{\partial X^2}\left(B^*_{11}\bar{N}^T_x + B^*_{21}\bar{N}^T_y\right) + 2\frac{\partial^2}{\partial X\partial Y}\left(B^*_{16}\bar{N}^T_x + B^*_{26}\bar{N}^T_y + B^*_{66}\bar{N}^T_{xy}\right) + \frac{\partial^2}{\partial Y^2}\left(B^*_{12}\bar{N}^T_x + B^*_{22}\bar{N}^T_y\right)\\
\tilde{L}_{14}(\bar{M}^T) &= \frac{\partial^2}{\partial X^2}\left(\bar{M}^T_x\right) + 2\frac{\partial^2}{\partial X\partial Y}\left(\bar{M}^T_{xy}\right) + \frac{\partial^2}{\partial Y^2}\left(\bar{M}^T_y\right)\\
\tilde{L}_{21}() &= A^*_{22}\frac{\partial^4}{\partial X^4} - 2A^*_{26}\frac{\partial^4}{\partial X^3\partial Y} + (2A^*_{12} + A^*_{66})\frac{\partial^4}{\partial X^2\partial Y^2} - 2A^*_{16}\frac{\partial^4}{\partial X\partial Y^3} + A^*_{11}\frac{\partial^4}{\partial Y^4}\\
\tilde{L}_{23}(\bar{N}^T) &= \frac{\partial^2}{\partial X^2}\left(A^*_{12}\bar{N}^T_x + A^*_{22}\bar{N}^T_y\right) - \frac{\partial^2}{\partial X\partial Y}\left(A^*_{16}\bar{N}^T_x + A^*_{26}\bar{N}^T_y + A^*_{66}\bar{N}^T_{xy}\right) + \frac{\partial^2}{\partial Y^2}\left(A^*_{11}\bar{N}^T_x + A^*_{12}\bar{N}^T_y\right)\\
\tilde{L}() &= \frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial Y^2} - 2\frac{\partial^2}{\partial X\partial Y}\frac{\partial^2}{\partial X\partial Y} + \frac{\partial^2}{\partial Y^2}\frac{\partial^2}{\partial X^2}
\end{aligned} \tag{4}$$

Note that the geometric nonlinearity in the von Kármán sense is given in terms of $\tilde{L}()$ in Eqs. (2) and (3).

Equations (2) and (3) are remarkable not only for the coupling between transverse bending and in-plane stretching, which is given in terms of B^*_{ij} $(i, j = 1, 2, 6)$, but also for the flexural/twist and extension/twist coupling indicated by D^*_{16}, D^*_{26}, A^*_{16}, and A^*_{26}. Note that for the cross-ply laminated cylindrical shells (using only 0 and 90 plies) all termsA^*_{16}, A^*_{26}, D^*_{16}, and D^*_{26} are zero-valued.

It is assumed that E_{11}, E_{22}, G_{12}, α_{11}, and α_{22} are functions of temperature, but Poisson's ratio ν_{12} depends weakly on temperature change and is assumed to be a constant. Now all the reduced stiffness matrices $[A^*_{ij}]$, $[B^*_{ij}]$, and $[D^*_{ij}]$ $(i, j = 1, 2, 6)$ are functions of temperature, defined as $\mathbf{A}^* = \mathbf{A}^{-1}$, $\mathbf{B}^* = -\mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{D}^* = \mathbf{D} - \mathbf{B}\mathrm{A}^{-1}\mathbf{B}$, where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{D}$ are defined by

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N}\int_{t_{k-1}}^{t_k}(\bar{Q}_{ij})_k(1, Z, Z^2)\,\mathrm{d}Z \qquad (i, j = 1, 2, 6) \tag{5}$$

in which $\bar{Q}_{ij}$ are the transformed elastic constants, defined by

$$\begin{bmatrix}\bar{Q}_{11}\\ \bar{Q}_{12}\\ \bar{Q}_{22}\\ \bar{Q}_{16}\\ \bar{Q}_{26}\\ \bar{Q}_{66}\end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2\\ c^2s^2 & c^4 + s^4 & c^2s^2 & -4c^2s^2\\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2\\ c^3s & cs^3 - c^3s & -cs^3 & -2cs(c^2 - s^2)\\ cs^3 & c^3s - cs^3 & -c^3s & 2cs(c^2 - s^2)\\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2 - s^2)^2\end{bmatrix}\begin{bmatrix}Q_{11}\\ Q_{12}\\ Q_{22}\\ Q_{66}\end{bmatrix} \tag{6a}$$

where

$$Q_{11} = \frac{E_{11}(T)}{(1 - \nu_{12}\nu_{21})}, \qquad Q_{22} = \frac{E_{22}(T)}{(1 - \nu_{12}\nu_{21})}$$
$$Q_{12} = \frac{\nu_{21}E_{11}(T)}{(1 - \nu_{12}\nu_{21})}, \qquad Q_{66} = G_{12}(T) \tag{6b}$$

E_{11}, E_{22}, G_{12}, ν_{12}, and ν_{21} have their usual meanings, and

$$c = \cos\theta, \qquad s = \sin\theta \tag{6c}$$

where θ is the lamination angle with respect to the shell X axis.

In the preceding equations, $\bar{N}^T$ and $\bar{M}^T$ are the forces and moments caused by elevated temperature and are defined by

$$\begin{bmatrix}\bar{N}^T_x & \bar{M}^T_x\\ \bar{N}^T_y & \bar{M}^T_y\\ \bar{N}^T_{xy} & \bar{M}^T_{xy}\end{bmatrix} = \sum_{k=1}^{N}\int_{t_{k-1}}^{t_k}\begin{bmatrix}A_x\\ A_y\\ A_{xy}\end{bmatrix}_k(1, Z)\Delta T\,\mathrm{d}Z \tag{7}$$

where $\Delta T = T - T_0$ is temperature rise from the reference temperature T_0 at which there are no thermal strains, and

$$\begin{bmatrix}A_x\\ A_y\\ A_{xy}\end{bmatrix} = -\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}c^2 & s^2\\ s^2 & c^2\\ 2cs & -2cs\end{bmatrix}\begin{bmatrix}\alpha_{11}(T)\\ \alpha_{22}(T)\end{bmatrix} \tag{8}$$

where α_{11} and α_{22} are the thermal expansion coefficients in the longitudinal and transverse directions.

The two end edges of the shell are assumed to be simply supported or clamped and to be restrained against expansion longitudinally while temperature is increased steadily, so that the boundary conditions are $(X = 0, L)$

Simply supported:

$$\bar{W} = 0, \qquad \bar{M}_x = 0 \tag{9a}$$

Clamped:

$$\bar{W} = \bar{W}_{,x} = 0 \tag{9b}$$

$$\bar{U} = 0 \tag{9c}$$

where

$$\bar{M}_x = -B^*_{21}\frac{\partial^2\bar{F}}{\partial X^2} - B^*_{11}\frac{\partial^2\bar{F}}{\partial Y^2} + B^*_{61}\frac{\partial^2\bar{F}}{\partial X\partial Y} - D^*_{11}\frac{\partial^2\bar{W}}{\partial X^2} - D^*_{12}\frac{\partial^2\bar{W}}{\partial Y^2} - 2D^*_{16}\frac{\partial^2\bar{W}}{\partial X\partial Y} \quad (10)$$

is the bending moment. Also, we have the closed (or periodicity) condition

$$\int_0^{2\pi R}\frac{\partial\bar{V}}{\partial Y}\,\mathrm{d}Y = 0 \quad (11a)$$

or

$$\int_0^{2\pi R}\left[A^*_{22}\frac{\partial^2\bar{F}}{\partial X^2} + A^*_{12}\frac{\partial^2\bar{F}}{\partial Y^2} - A^*_{26}\frac{\partial^2\bar{F}}{\partial X\partial Y} - \left(B^*_{21}\frac{\partial^2\bar{W}}{\partial X^2} + B^*_{22}\frac{\partial^2\bar{W}}{\partial Y^2} + 2B^*_{26}\frac{\partial^2\bar{W}}{\partial X\partial Y}\right) + \frac{\bar{W}}{R} - \frac{1}{2}\left(\frac{\partial\bar{W}}{\partial Y}\right)^2 - \frac{\partial\bar{W}}{\partial Y}\frac{\partial\bar{W}^*}{\partial Y} - (A^*_{12}\bar{N}^T_x + A^*_{22}\bar{N}^T_y + A^*_{26}\bar{N}^T_{xy})\right]\mathrm{d}Y = 0 \quad (11b)$$

Because of Eq. (11), the in-plane boundary condition $\bar{V} = 0$ (at $X = 0, L$) is not needed in Eq. (9).

The average end-shortening relationship is defined as

$$\frac{\Delta_x}{L} = -\frac{1}{2\pi RL}\int_0^{2\pi R}\int_0^L\frac{\partial\bar{U}}{\partial X}\,\mathrm{d}X\,\mathrm{d}Y = -\frac{1}{2\pi RL}\int_0^{2\pi R}\int_0^L\left[A^*_{11}\frac{\partial^2\bar{F}}{\partial Y^2} + A^*_{12}\frac{\partial^2\bar{F}}{\partial X^2} - A^*_{16}\frac{\partial^2\bar{F}}{\partial X\partial Y} - \left(B^*_{11}\frac{\partial^2\bar{W}}{\partial X^2} + B^*_{12}\frac{\partial^2\bar{W}}{\partial Y^2} + 2B^*_{16}\frac{\partial^2\bar{W}}{\partial X\partial Y}\right) - \frac{1}{2}\left(\frac{\partial\bar{W}}{\partial X}\right)^2 - \frac{\partial\bar{W}}{\partial X}\frac{\partial\bar{W}^*}{\partial X} - (A^*_{11}\bar{N}^T_x + A^*_{12}\bar{N}^T_y + A^*_{16}\bar{N}^T_{xy})\right]\mathrm{d}X\,\mathrm{d}Y \quad (12)$$

where Δ_x is shell end-shortening displacement in the X direction. Note that the boundary condition (9c) implies that the end-shortening vanishes.

B. Boundary-Layer-Type Equations

Before proceeding, it is convenient to first define the following dimensionless quantities:

$$x = \pi X/L, \qquad y = Y/R, \qquad \beta = L/\pi R, \qquad \bar{Z} = L^2/Rt$$
$$\varepsilon = (\pi^2 R/L^2)[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}$$
$$(W,\ W^*) = \varepsilon(\bar{W},\ \bar{W}^*)/[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}$$
$$F = \varepsilon^2\overline{F}/[D^*_{11}D^*_{22}]^{1/2}$$
$$(\gamma_{11}, \gamma_{12}, \gamma_{13}, \gamma^2_{14}) = (D^*_{16}, D^*_{12} + 2D^*_{66}, D^*_{26}, D^*_{22})/D^*_{11}$$
$$(\gamma_{21}, \gamma_{22}, \gamma_{23}, \gamma^2_{24}) = (A^*_{26}, A^*_{12} + \tfrac{1}{2}A^*_{66}, A^*_{16}, A^*_{11})/A^*_{22}$$
$$\gamma_5 = -A^*_{12}/A^*_{22}, \qquad (\gamma_{30}, \gamma_{31}, \gamma_{32}, \gamma_{33}, \gamma_{34}) = (B^*_{21}, 2B^*_{26} - B^*_{61}, B^*_{11} + B^*_{22} - 2B^*_{66}, 2B^*_{16} - B^*_{62}, B^*_{12})/[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}$$
$$(\gamma_{311}, \gamma_{322}, \gamma_{316}, \gamma_{326}) = (B^*_{11}, B^*_{22}, B^*_{16}, B^*_{26})/[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}$$
$$(M_x, M_y) = \varepsilon^2(\bar{M}_x, \bar{M}_y)(L^2/\pi^2)/D^*_{11}[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}$$
$$(\gamma_{T1}, \gamma_{T2}, \gamma_{T3}) = (A^T_x, A^T_y, A^T_{xy})R[A^*_{11}A^*_{22}/D^*_{11}D^*_{22}]^{1/4}$$
$$\lambda_T = \alpha_0\Delta T, \qquad \delta_x = \left(\frac{\Delta_x}{L}\right)\frac{R}{2[D^*_{11}D^*_{22}A^*_{11}A^*_{22}]^{1/4}} \quad (13)$$

where α_0 is an arbitrary reference value, and

$$\alpha_{11} = a_{11}\alpha_0, \qquad \alpha_{22} = a_{22}\alpha_0 \quad (14)$$

Also let

$$\begin{bmatrix} A^T_x \\ A^T_y \\ A^T_{xy}\end{bmatrix}\Delta T = -\sum_{k=1}^{N}\int_{t_{k-1}}^{t_k}\begin{bmatrix} A_x \\ A_y \\ A_{xy}\end{bmatrix}_k \Delta T\,\mathrm{d}Z \quad (15)$$

The nonlinear equations (2) and (3) may then be written in dimensionless form as

$$\varepsilon^2 L_{11}(W) + \varepsilon\gamma_{14}L_{12}(F) - \gamma_{14}F_{,xx} = \gamma_{14}\beta^2 L(W + W^*, F) \quad (16)$$

$$L_{21}(F) - \varepsilon\gamma_{24}L_{22}(W) + \gamma_{24}W_{,xx} = -\frac{1}{2}\gamma_{24}\beta^2 L(W + 2W^*, W) \quad (17)$$

where

$$L_{11}() = \frac{\partial^4}{\partial x^4} + 4\gamma_{11}\beta\frac{\partial^4}{\partial x^3\partial y} + 2\gamma_{12}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} + 4\gamma_{13}\beta^3\frac{\partial^4}{\partial x\partial y^3} + \gamma^2_{14}\beta^4\frac{\partial^4}{\partial y^4}$$
$$L_{12}() = L_{22}() = \gamma_{30}\frac{\partial^4}{\partial x^4} + \gamma_{31}\beta\frac{\partial^4}{\partial x^3\partial y} + \gamma_{32}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} + \gamma_{33}\beta^3\frac{\partial^4}{\partial x\partial y^3} + \gamma_{34}\beta^4\frac{\partial^4}{\partial y^4}$$
$$L_{21}() = \frac{\partial^4}{\partial x^4} - 2\gamma_{21}\beta\frac{\partial^4}{\partial x^3\partial y} + 2\gamma_{22}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} - 2\gamma_{23}\beta^3\frac{\partial^4}{\partial x\partial y^3} + \gamma^2_{24}\beta^4\frac{\partial^4}{\partial y^4}$$
$$L() = \frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} - 2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y} + \frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2} \quad (18)$$

The boundary conditions expressed by Eq. (9) become ($x = 0, \pi$)

Simply supported:

$$W = M_x = 0 \quad (19a)$$

Clamped:

$$W = W_{,x} = 0 \quad (19b)$$

$$\delta_x = 0 \quad (19c)$$

and the closed condition expressed by Eq. (11b) becomes

$$\int_0^{2\pi}\left[\left(\frac{\partial^2 F}{\partial x^2} - \gamma_5\beta^2\frac{\partial^2 F}{\partial y^2} - \gamma_{21}\beta\frac{\partial^2 F}{\partial x\partial y}\right) - \varepsilon\gamma_{24}\left(\gamma_{30}\frac{\partial^2 W}{\partial x^2} + \gamma_{322}\beta^2\frac{\partial^2 W}{\partial y^2} + 2\gamma_{326}\beta\frac{\partial^2 W}{\partial x\partial y}\right) + \gamma_{24}W - \frac{1}{2}\gamma_{24}\beta^2\left(\frac{\partial W}{\partial y}\right)^2 - \gamma_{24}\beta^2\frac{\partial W}{\partial y}\frac{\partial W^*}{\partial y} + \varepsilon(\gamma_{T2} - \gamma_5\gamma_{T1} + \gamma_{21}\gamma_{T3})\lambda_T\right]dy = 0 \quad (20)$$

The unit end-shortening relationship becomes

$$\delta_x = -\frac{1}{4\pi^2\gamma_{24}}\varepsilon^{-1}\int_0^{2\pi}\int_0^{\pi}\left[\left(\gamma^2_{24}\beta^2\frac{\partial^2 F}{\partial y^2} - \gamma_5\frac{\partial^2 F}{\partial x^2} - \gamma_{23}\beta\frac{\partial^2 F}{\partial x\partial y}\right) - \varepsilon\gamma_{24}\left(\gamma_{311}\frac{\partial^2 W}{\partial x^2} + \gamma_{34}\beta^2\frac{\partial^2 W}{\partial y^2} + 2\gamma_{316}\beta\frac{\partial^2 W}{\partial x\partial y}\right) - \frac{1}{2}\gamma_{24}\left(\frac{\partial W}{\partial x}\right)^2 - \gamma_{24}\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x} + \varepsilon(\gamma^2_{24}\gamma_{T1} - \gamma_5\gamma_{T2} + \gamma_{23}\gamma_{T3})\lambda_T\right]\mathrm{d}x\,\mathrm{d}y \quad (21)$$

By virtue of the fact that ΔT is assumed to be uniform, the thermal coupling in Eqs. (2) and (3) vanishes, but the terms in ΔT affect Eqs. (20) and (21).

In Eq. (13), we introduce an important parameter ε. For most composite materials, $[D_{11}^* D_{22}^* A_{11}^* A_{22}^*]^{1/4} \cong 0.3t$. Furthermore, when $\bar{Z} = (L^2/Rt) > 2.96$, then from Eq. (13), $\varepsilon < 1$. In particular, for isotropic cylindrical shells, $\varepsilon = \pi^2/\bar{Z}_B\sqrt{12}$, where $\bar{Z}_B = (L^2/Rt)[1-\nu^2]^{1/2}$ is the Batdorf shell parameter, which should be greater than 2.85 in the case of classical linear buckling analysis [17]. In practice, the shell structure will have $\bar{Z} > 10$, so that we always have $\varepsilon \ll 1$. When $\varepsilon < 1$, Eqs. (16) and (17) are of the boundary-layer type, from which nonlinear prebuckling deformations, large deflections in the postbuckling range, and initial geometric imperfections of the shell can be considered simultaneously.

III. Solution Procedure

Having developed the theory, we are now in a position to solve Eqs. (16) and (17) with boundary condition (19) by means of a singular perturbation technique. The essence of this procedure, in the present case, is to assume that

$$W = w(x,y,\varepsilon) + \tilde{W}(x,\xi,y,\varepsilon) + \hat{W}(x,\zeta,y,\varepsilon)$$
$$F = f(x,y,\varepsilon) + \tilde{F}(x,\xi,y,\varepsilon) + \hat{F}(x,\zeta,y,\varepsilon) \quad (22)$$

where ε is a small perturbation parameter (provided $\bar{Z} > 2.96$), as defined in Eq. (13), and $w(x, y, \varepsilon)$ and $f(x, y, \varepsilon)$ are called the outer or regular solutions of the shell. $\tilde{W}(x, \xi, y, \varepsilon)$ $[\tilde{F}(x, \xi, y, \varepsilon)]$ and $\hat{W}(x, \zeta, y, \varepsilon)$ $[\hat{F}(x, \zeta, y, \varepsilon)]$ are the boundary-layer solutions near the $x = 0$ and $x = \pi$ edges, respectively, and ξ and ζ are the boundary-layer variables, defined as

$$\xi = x/\sqrt{\varepsilon}, \qquad \zeta = (\pi - x)/\sqrt{\varepsilon} \quad (23)$$

This means that for isotropic cylindrical shells, the width of the boundary layers is on the order of $\sqrt{Rt}$. In Eq. (22), the regular and boundary-layer solutions are taken in the forms of perturbation expansions as

$$w(x,y,\varepsilon) = \sum_{j=1} \varepsilon^j w_j(x,y), \qquad f(x,y,\varepsilon) = \sum_{j=0} \varepsilon^j f_j(x,y) \quad (24a)$$

$$\tilde{W}(x,\xi,y,\varepsilon) = \sum_{j=0} \varepsilon^{j+1} \tilde{W}_{j+1}(x,\xi,y),$$
$$\tilde{F}(x,\xi,y,\varepsilon) = \sum_{j=0} \varepsilon^{j+2} \tilde{F}_{j+2}(x,\xi,y) \quad (24b)$$

$$\hat{W}(x,\zeta,y,\varepsilon) = \sum_{j=0} \varepsilon^{j+1} \hat{W}_{j+1}(x,\zeta,y),$$
$$\hat{F}(x,\zeta,y,\varepsilon) = \sum_{j=0} \varepsilon^{j+2} \hat{F}_{j+2}(x,\zeta,y) \quad (24c)$$

Because the flexural/twist coupling is included in $L_{11}()$ and the extension/twist coupling is included in $L_{21}()$, $(\sin mx \sin ny)$ is no longer the solution of Eqs. (16) and (17). As in the case of axial compression [14], we assume the initial buckling mode to have the form

$$w_2(x,y) = A_{00}^{(2)} + A_{11}^{(2)} \sin mx \sin ny + a_{11}^{(2)} \cos mx \cos ny + A_{02}^{(2)} \cos 2ny \quad (25)$$

and the initial geometric imperfection is assumed to have the form

$$W^*(x,y,\varepsilon) = \varepsilon^2 \mu \left[A_{11}^{(2)} \sin mx \sin ny + a_{11}^{(2)} \cos mx \cos ny\right] \quad (26)$$

where μ is the imperfection parameter.

It is noted that Eq. (1) can be rewritten as $w_1(\sin mx \cos ny - \cos mx \sin ny)$, analogous to the second and third components of Eq. (25) but with the same amplitude.

Substituting Eqs. (22–24) into Eqs. (16) and (17) and collecting terms of the same order of ε, one obtains three sets of perturbation equations for the regular and boundary-layer solutions, respectively.

Using Eqs. (25) and (26) to solve these perturbation equations of each order and matching the regular solutions with the boundary-layer solutions at each end of the shell, details of which can be found in Shen [14], the asymptotic solutions satisfying the clamped boundary conditions are constructed as

$$\begin{aligned}
W = {} & \varepsilon\bigg[A_{00}^{(1)} - A_{00}^{(1)}\left(\cos\phi\frac{x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right) \\
& - A_{00}^{(1)}\left(\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\bigg] \\
& + \varepsilon^2\big[A_{00}^{(2)} + A_{11}^{(2)}\sin mx\sin ny + a_{11}^{(2)}\cos mx\cos ny \\
& + A_{02}^{(2)}\cos 2ny - (A_{00}^{(2)} + a_{11}^{(2)}\cos ny \\
& + A_{02}^{(2)}\cos 2ny)\left(\cos\phi\frac{x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right) \\
& - (A_{00}^{(2)} + a_{11}^{(2)}\cos ny + A_{02}^{(2)}\cos 2ny)\bigg(\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}} \\
& + \frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\bigg)\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\big] \\
& + \varepsilon^3\big[A_{00}^{(3)} + A_{11}^{(3)}\sin mx\sin ny + a_{11}^{(3)}\cos mx\cos ny \\
& + A_{02}^{(3)}\cos 2ny\big] + \varepsilon^4\big[A_{00}^{(4)} + A_{11}^{(4)}\sin mx\sin ny \\
& + a_{11}^{(4)}\cos mx\cos ny + A_{20}^{(4)}\cos 2mx + A_{02}^{(4)}\cos 2ny \\
& + A_{13}^{(4)}\sin mx\sin 3ny + a_{13}^{(4)}\cos mx\cos 3ny \\
& + A_{04}^{(4)}\cos 4ny\big] + O(\varepsilon^5)
\end{aligned} \quad (27)$$

$$\begin{aligned}
F = {} & -B_{00}^{(0)}\frac{y^2}{2} - b_{00}^{(0)}xy + \varepsilon\left[-B_{00}^{(1)}\frac{y^2}{2} - b_{00}^{(1)}xy\right] \\
& + \varepsilon^2\bigg[-B_{00}^{(2)}\frac{y^2}{2} - b_{00}^{(2)}xy + B_{11}^{(2)}\sin mx\sin ny \\
& + A_{00}^{(1)}\left(b_{01}^{(2)}\cos\phi\frac{x}{\sqrt{\varepsilon}} + b_{10}^{(2)}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right) \\
& + A_{00}^{(1)}\left(b_{01}^{(2)}\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}} + b_{10}^{(2)}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\bigg] \\
& + \varepsilon^3\bigg[-B_{00}^{(3)}\frac{y^2}{2} - b_{00}^{(3)}xy + B_{02}^{(3)}\cos 2ny \\
& + (A_{00}^{(2)} + a_{11}^{(2)}\cos ny + A_{02}^{(2)}\cos 2ny) \\
& \times\left(b_{01}^{(3)}\cos\phi\frac{x}{\sqrt{\varepsilon}} + b_{10}^{(3)}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right) \\
& + (A_{00}^{(2)} + a_{11}^{(2)}\cos ny + A_{02}^{(2)}\cos 2ny) \\
& \times\left(b_{01}^{(3)}\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}} + b_{10}^{(3)}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\bigg] \\
& + \varepsilon^4\bigg[-B_{00}^{(4)}\frac{y^2}{2} - b_{00}^{(4)}xy + B_{20}^{(4)}\cos 2mx + B_{02}^{(4)}\cos 2ny \\
& + B_{13}^{(4)}\sin mx\sin 3ny + b_{13}^{(4)}\cos mx\cos 3ny\bigg] + O(\varepsilon^5)
\end{aligned} \quad (28)$$

Note that because of Eq. (27), the prebuckling deformation of the shell is nonlinear, and all of the coefficients in Eqs. (27) and (28) are related and can be expressed in terms of $A_{11}^{(2)}$, but for the sake of brevity, the detailed expressions are not shown, although α and ϕ are given in detail in the Appendix.

Next, upon substitution of Eqs. (27) and (28) into boundary condition (19c) and into closed conditions (20) and (21), the thermal postbuckling equilibrium path can be written as

$$\lambda_T = C_{11}[\lambda_T^{(0)} - \lambda_T^{(2)}(A_{11}^{(2)}\varepsilon)^2 + \lambda_T^{(4)}(A_{11}^{(2)}\varepsilon)^4 + \cdots] \quad (29)$$

In Eq. (29), $(A_{11}^{(2)}\varepsilon)$ is taken as the second perturbation parameter relating to the dimensionless maximum deflection. From Eq. (27), by taking $(x, y) = (\pi/2m, \pi/2n)$, one has

$$A_{11}^{(2)}\varepsilon = W_m - \Theta_2 W_m^2 + \cdots \quad (30a)$$

where W_m is the dimensionless form of the maximum deflection of the shell that can be written as

$$W_m = \frac{1}{C_3}\left[\frac{t}{[D_{11}^* D_{22}^* A_{11}^* A_{22}^*]^{1/4}}\frac{\bar{W}}{t} + \Theta_1\right] \quad (30b)$$

All symbols used in Eqs. (29) and (30) are also described in detail in the Appendix. It is noted that $\lambda_T^{(i)}$ $(i = 0, 2, \ldots)$ are all functions of temperature. It is evident that from Eq. (28) there exists a compressive stress along with an associate shear stress when the shell is subjected to heating. Such a shear stress, no matter how small it is, will affect the thermal buckling and postbuckling behavior of an anisotropic laminated cylindrical shell, as shown in Eqs. (A1) and (A3) in the Appendix, but it is missing in all the previously analyses.

IV. Numerical Results and Discussion

Numerical results are presented in this section for perfect and imperfect anisotropic laminated cylindrical shells with extension/twist, extension/flexural, and flexural/twist couplings, for which the outmost layer is the first mentioned orientation.

Graphite/epoxy composite material was selected for the fiber-reinforced orthotropic layers, the material properties of which are assumed to be linear functions of temperature change; that is,

$$E_{11}(T) = E_{110}(1 + E_{111}\Delta T), \quad E_{22}(T) = E_{220}(1 + E_{221}\Delta T)$$
$$G_{12}(T) = G_{120}(1 + G_{121}\Delta T) \quad \alpha_{11}(T) = \alpha_{110}(1 + \alpha_{111}\Delta T) \quad (31)$$
$$\alpha_{22}(T) = \alpha_{220}(1 + \alpha_{221}\Delta T)$$

where $E_{110}, E_{220}, G_{120}, \alpha_{110}, \alpha_{220}, E_{111}, E_{221}, G_{121}, \alpha_{111}$, and α_{221} are constants. Poisson's ratio depends weakly on temperature change and is assumed to be a constant. Typical values adopted, as given in Oh et al. [18], are $E_{110} = 150$ GPa, $E_{220} = 9.0$ GPa, $G_{120} = 7.1$ GPa, $\alpha_{110} = 1.1 \times 10^{-6}/°C$, $\alpha_{220} = 25.2 \times 10^{-6}/°C$, $\nu_{12} = 0.3$, $E_{111} = -0.0005$, $E_{221} = G_{121} = -0.0002$, and $\alpha_{111} = \alpha_{221} = 0.0005$.

To obtain numerical results, it is necessary to solve Eq. (29) by an iterative numerical procedure with the following steps:

1) Begin with $\bar{W}/t = 0$.

2) Assume that elastic constants and the thermal expansion coefficients are constant (i.e., at $\Delta T = 0$). The thermal buckling load for the shell of temperature-independent material is obtained.

3) Use the temperature determined in the previous step [the temperature-dependent material properties may be decided from Eq. (31)] and the thermal buckling load is obtained again.

4) Repeat step 3 until the thermal buckling temperature converges.

5) Specify the new value of $\bar{W}/t$, and steps 2–4 are repeated until the thermal postbuckling temperature converges.

We first examine the effect of the radius-to-thickness ratio ($R/t = 200$, 300, 400, and 500) on the buckling temperature of (0/90/0) laminated cylindrical shells subjected to uniform temperature rise. The buckling loads are calculated and are compared in Table 1 with the FEM results of Thangaratnam [19], Ganesan and Kadoli [7], and Patel et al. [20]. In computation, $L/R = 0.5$, $E_{11}/E_{22} = 10$, $G_{12}/E_{22} = 0.5$, $\nu_{12} = 0.25$, $\alpha_{22}/\alpha_{11} = 2$, and $\alpha_{11} = 1.0 \times 10^{-6}/°C$. It can be seen that the present results are in good agreement with those of Patel et al. [20]. In contrast, Thangaratnam [19] gave poor FEM results in all four cases. In Table 1, (m, n) represents the buckling mode, which determines the number of half-waves in the X direction and full waves in the Y direction. In the usual postbuckling analysis, the buckling mode of the shell is assumed to remain unchanged. In reality, mode changes are possible in the deep

Table 1 Comparisons of buckling temperature ΔT_{cr} (°C) for a perfect (0/90/0) laminated cylindrical shell subjected to uniform temperature rise ($L/R = 0.5$, $E_{11}/E_{22} = 10$, $G_{12}/E_{22} = 0.5$, $\nu_{12} = 0.25$, $\alpha_{22}/\alpha_{11} = 2$, and $\alpha_{11} = 1.0 \times 10^{-6}/°C$)

	ΔT_{cr}, °C			
R/t	Thangaratnam [19]	Ganesan and Kadoli [7]	Patel et al. [20]	Present
200	1304.298 (11)[a]	1250 (12)	1197.8608 (11)	1198.7690 (3,11)[b]
300	912.434 (13)	835.8 (14)	843.1547 (14)	836.9448 (3,13)
400	659.610 (18)	631 (17)	594 0274 (17)	598.8117 (5,17)
500	514.745 (19)	507 (19)	478.6606 (18)	477.1969 (5,18)

[a]The number in parentheses indicates the full wave number in the circumferential direction.
[b]The numbers in parentheses indicate the buckling mode (m, n).

Table 2 Comparisons of buckling temperature/shear stress for perfect laminated cylindrical shells under uniform temperature rise ($R/t = 200$, $t = 1.0$ mm, $E_{110} = 150.0$ GPa, $E_{220} = 9.0$ GPa, $G_{120} = 7.1$ GPa, $\nu_{12} = 0.3$, $\alpha_{110} = 1.1 \times 10^{-6}/°C$, $\alpha_{220} = 25.2 \times 10^{-6}/°C$, $E_{111} = -0.0005$, $E_{221} = G_{121} = -0.0002$, and $\alpha_{111} = \alpha_{221} = 0.0005$)

			$\bar{Z} = 200$		$\bar{Z} = 500$		$\bar{Z} = 800$	
Layup	A_{i6}^*	D_{i6}^*	ΔT_{cr}, °C	τ_s, MPa	ΔT_{cr}, °C	τ_s, MPa	ΔT_{cr}, °C	τ_s, MPa
			TID					
$(0/90)_{2S}$	0	0	423.3466	0	425.8437	0	424.1917	0
$(\pm 45)_{2S}$	~e − 16	~e − 0	1534.531	−4.09	1584.72	−2.93	1655.969	−4.09
$(-45_2/-30_2/60_2/15_2)_T$	~e − 9	~e − 1	748.7549	+0.39	769.6483	+0.42	792.4609	+0.63
$(15_2/60_2/-30_2/-45_2)_T$	~e − 9	~e − 1	917.9862	+2.97	930.8359	+1.65	962.4398	+2.96
			TD					
$(0/90)_{2S}$	0	0	355.1019	0	356.6692	0	355.7793	0
$(\pm 45)_{2S}$	~e − 16	~e − 0	804.4615	−2.39	813.2157	−1.70	830.7313	−2.33
$(-45_2/-30_2/60_2/15_2)_T$	~e − 9	~e − 1	551.7434	+0.35	561.4419	+0.40	574.3632	+0.58
$(15_2/60_2/-30_2/-45_2)_T$	~e − 9	~e − 1	624.1876	+2.13	635.7204	+1.16	644.0468	+2.10

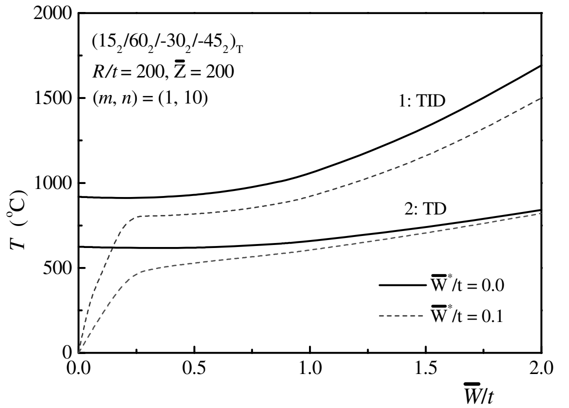

Fig. 1 Effect of material properties on the thermal postbuckling behavior of the cylindrical shell.

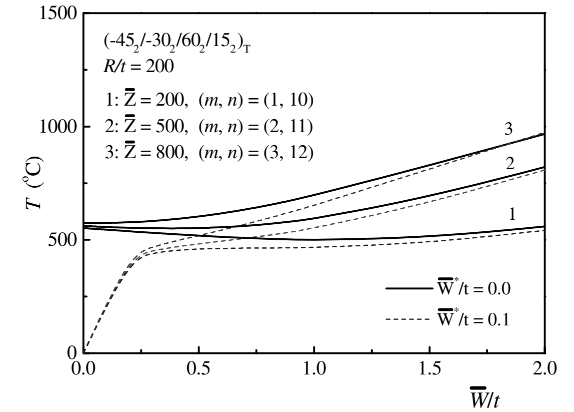

Fig. 3 Effect of shell geometric parameter on the thermal postbuckling behavior of cylindrical shells.

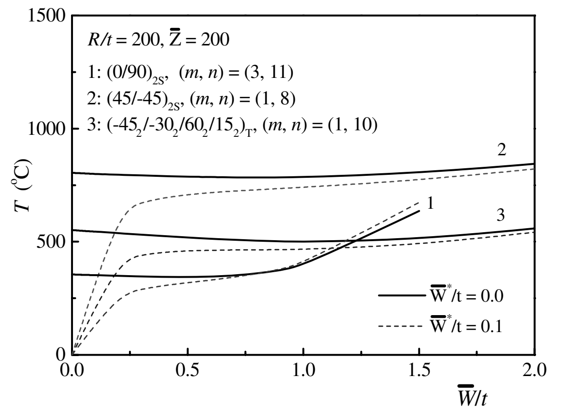

Fig. 2 Anisotropic effect on the thermal postbuckling behavior of cylindrical shells.

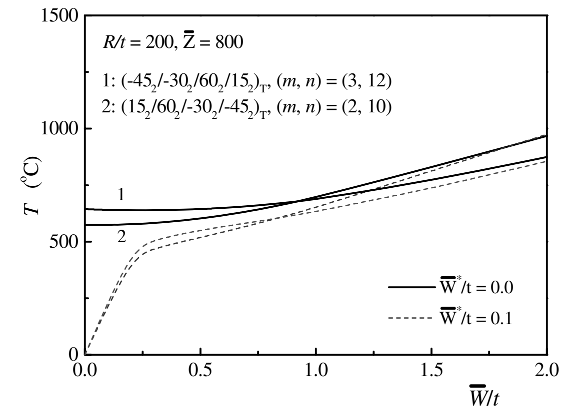

Fig. 4 Comparisons of thermal postbuckling behavior of cylindrical shells.

postbuckling range; however, such mode changes are not considered in the present study.

In addition, the thermal postbuckling load-deflection curves for a $(0/90)_S$ laminated cylindrical shell under uniform temperature rise were examined by a comparison study given by Patel et al. [21]. The results were found to be in good agreement, thus verifying the reliability and accuracy of the present method. Note that in these two comparison studies, the material properties were assumed to be independent of temperature.

A parametric study intended to supply information on the thermal postbuckling response of anisotropic laminated cylindrical shells subjected to uniform temperature rise was undertaken. Typical results are shown in Table 2 and Figs. 1–4. For these examples, $R/t = 200$, all plies are of equal thickness, and the total thickness of the shell is $t = 1.0$ mm.

Table 2 gives the buckling temperature ΔT_{cr} (°C) and associated shear stress τ_s (MPa) for perfect, $(0/90)_{2S}$ symmetric cross-ply, $(\pm 45)_{2S}$ symmetric angle-ply, and $(-45_2/-30_2/60_2/15_2)_T$ and $(15_2/60_2/-30_2/-45_2)_T$ laminated cylindrical shells with different values of shell parameters ($\bar{Z} = 200$, 500, and 800) subjected to uniform temperature rise. The extension/twist and flexural/twist couplings denoted by A^*_{i6} and D^*_{i6} ($i = 1, 2$) are also listed in Table 2. As expected, the shear stresses τ_s are zero-valued for the $(0/90)_{2S}$ shell because of no couplings. Because of the geometrical symmetry, the extension/twist couplings A^*_{i6} ($i = 1, 2$) are approximately zero-valued for the $(\pm 45)_{2S}$ shell, and the shear stress τ_s is negative. It is noted that the $(15_2/60_2/-30_2/-45_2)_T$ shell has the reverse stacking sequence to the $(-45_2/-30_2/60_2/15_2)_T$ shell, and the shear stress τ_s is positive. In Table 2, TD represents material properties for graphite/epoxy orthotropic layers that are temperature-dependent. TID represents material properties for graphite/epoxy orthotropic layers are temperature-independent [i.e., $E_{111} = E_{221} = G_{121} = \alpha_{111} = \alpha_{221} = 0$ in Eq. (31)]. It can be seen that the buckling temperature decreases when the temperature dependency is put into consideration. The percentage decrease is about −16, −47, −27, and −32% for these four shells with the same geometric parameters. It is noted that the buckling temperature for a $(\pm 45)_{2S}$ shell is very large, and the material of the shell wall may be destroyed before attaining such a large temperature.

Figure 1 shows the thermal postbuckling load-deflection curves for perfect and imperfect $(15_2/60_2/-30_2/-45_2)_T$ laminated cylindrical shells with $\bar{Z} = 200$ under two cases of thermoelastic material properties (i.e., TID and TD). It can be seen that the thermal postbuckling equilibrium path becomes lower when the temperature-dependent properties are taken into account. Therefore, in the following parametric study, only the TD case is considered.

Figure 2 shows the thermal postbuckling load-deflection curves for $(0/90)_{2S}$, $(\pm 45)_{2S}$, and $(-45_2/-30_2/60_2/15_2)_T$ laminated cylindrical shells with $\bar{Z} = 200$ when subjected to a uniform temperature rise under the TD case. It can be seen that these three shells will have different thermal postbuckling responses under the same loading condition. The $(0/90)_{2S}$ shell has a lower buckling temperature, but the postbuckling load increases sharply when deflection $\bar{W}/t > 1.0$. In contrast, the thermal postbuckling load-deflection curves for $(\pm 45)_{2S}$ and $(-45_2/-30_2/60_2/15_2)_T$ laminated cylindrical shells are relatively smooth.

Figure 3 shows the effect of the shell geometric parameter ($\bar{Z} = 200$, 500, and 800) on the thermal postbuckling behavior of $(-45_2/-30_2/60_2/15_2)_T$ laminated cylindrical shells under uniform temperature rise. It can be seen that the shell becomes stiffer as the shell geometric parameter $\bar{Z}$ is increased, which confirms the finding of Shen [5] for piezolaminated cylindrical shells. This is due to the fact that longitudinal expansion could not occur when temperature is increased steadily.

Figure 4 compares the thermal postbuckling load-deflection curves for $(-45_2/-30_2/60_2/15_2)_T$ and $(15_2/60_2/-30_2/-45_2)_T$ laminated cylindrical shells with $\bar{Z}=800$ under uniform temperature rise. The results show that the $(15_2/60_2/-30_2/-45_2)_T$ shell has a lower buckling temperature but has higher postbuckling loads than the $(-45_2/-30_2/60_2/15_2)_T$ shell when $\bar{W}/t > 1.0$.

It is noted that in all these figures, $\bar{W}^*/t$ denotes the dimensionless maximum initial geometric imperfection of the shell. From Figs. 1–4, it can be seen that the thermal postbuckling equilibrium path is stable or weakly unstable, and the shell structure is virtually imperfection-insensitive for both TID and TD cases. Such a phenomenon was previously reported in Shen [4–6], and Patel et al. [21,22].

V. Conclusions

A fully thermal postbuckling analysis is presented based on classical shell theory with a von-Kármán–Donnell-type of kinematic nonlinearity. Material properties are assumed to be functions of temperature. The boundary-layer theory of shell buckling was extended to the case of anisotropic cylindrical shells subjected to a uniform temperature rise. A singular perturbation technique is employed to determine buckling temperature and postbuckling load-deflection curves. The solutions presented give an insight into interaction between the extension/twist, extension/flexural, and flexural/twist couplings. The present study raises some new issues in the field of shell thermal buckling, both for the understanding of buckling phenomena and for the relational design of shells by engineers against buckling. The new finding is that there exists a compressive stress along with an associate shear stress when the anisotropic shell is subjected to heating. The results show that the anisotropy has a significant effect on the buckling temperature and the postbuckling behavior of laminated cylindrical shells. The results also confirm that the thermal postbuckling equilibrium path is stable or weakly unstable and the shell structure is virtually imperfection-insensitive.

Appendix: Coefficients

In Eqs. (29) and (30),

$$\Theta_1 = \frac{2\gamma_5}{\gamma_{24}}\lambda_p^{(0)} + \frac{\gamma_{21}}{\gamma_{24}}\lambda_s^{(0)} + \frac{C_{pt}}{\gamma_{24} g_t}(\gamma_{T2} - \gamma_5\gamma_{T1} + \gamma_{21}\gamma_{T3})\lambda_T^{(0)}$$

$$\Theta_2 = \frac{1}{C_3}\left\{\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^4(1+\mu)}{16n^2\beta^2}\varepsilon^{-1} - \left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^2(g_{21}g_{31}-g_{22}g_{32})}{16n^2\beta^2 g_{21}^2} + \left(\frac{\gamma_{34}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^2(g_{21}^2-g_{22}^2)}{32n^2\beta^2 g_{21}^2}(1+2\mu)\right.$$
$$\left. + \frac{2\gamma_5}{\gamma_{24}}\lambda_p^{(2)} + \frac{\gamma_{21}}{\gamma_{24}}\lambda_s^{(2)} + \frac{C_{pt}}{\gamma_{24} g_t}(\gamma_{T2} - \gamma_5\gamma_{T1} + \gamma_{21}\gamma_{T3})\lambda_T^{(2)}\right\}$$

$$\lambda_T^{(0)} = 2\lambda_p^{(0)} - \frac{C_{st}}{C_{pt}}\lambda_s^{(0)}, \qquad \lambda_T^{(2)} = 2\lambda_p^{(2)} - \frac{C_{st}}{C_{pt}}\lambda_s^{(2)} - \frac{2\gamma_{24}}{C_{pt}}\delta_x^{(2)}, \qquad \lambda_T^{(4)} = 2\lambda_p^{(4)} - \frac{C_{st}}{C_{pt}}\lambda_s^{(4)} + \frac{2\gamma_{24}}{C_{pt}}\delta_x^{(4)} \tag{A1}$$

$$\lambda_p^{(0)} = \frac{1}{2}\left\{\frac{\gamma_{24}m^2 g_{21}}{(g_{21}^2-g_{22}^2)(1+\mu)}\varepsilon^{-1} + \gamma_{24}\frac{g_{21}g_{31}+g_{22}g_{32}}{g_{21}^2-g_{22}^2}\frac{(2+\mu)}{(1+\mu)^2} + \frac{1}{(1+\mu)m^2}\left[\frac{g_{11}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{21}(g_{31}^2+g_{32}^2)+2g_{22}g_{31}g_{32}}{g_{21}^2-g_{22}^2}\right]\varepsilon\right.$$
$$- \frac{\mu}{m^4(1+\mu)^2}\left[\frac{g_{11}g_{31}+g_{12}g_{32}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{21}g_{31}(g_{31}^2+3g_{32}^2)+g_{22}g_{32}(3g_{31}^2+g_{32}^2)}{g_{21}^2-g_{22}^2}\right]\varepsilon^2$$
$$\left. + \frac{\mu^2}{m^6(1+\mu)^3}\left[\frac{g_{11}(g_{31}^2+g_{32}^2)+2g_{12}g_{31}g_{32}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{31}(g_{21}g_{31}+g_{22}g_{32})(g_{31}^2+3g_{32}^2)+g_{32}(g_{21}g_{32}+g_{22}g_{31})(3g_{31}^2+g_{32}^2)}{g_{21}^2-g_{22}^2}\right]\varepsilon^3\right\}$$

$$\lambda_p^{(2)} = \frac{1}{2}\Bigg(\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^6(g_{21}^2+g_{22}^2)}{4(g_{21}^2-g_{22}^2)^2}\varepsilon^{-1} + \left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^4 g_{22}}{2(g_{21}^2-g_{22}^2)^3 g_{21}(1+\mu)}[g_{32}(2g_{21}^2+g_{22}^2)-g_{21}g_{22}g_{31}]$$
$$+ \left(\frac{\gamma_{34}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^4(g_{21}^2+g_{22}^2)}{8(g_{21}^2-g_{22}^2)g_{21}(1+\mu)}[(1+2\mu)+(1+\mu)^2] - \left(\frac{\gamma_{14}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^2}{16}\frac{g_{21}^2+g_{22}^2}{g_{21}^2}(1+2\mu)\varepsilon$$
$$+ \frac{1}{4}\frac{\gamma_{24}m^2n^4\beta^4}{g_{21}(1+\mu)}\frac{g_{21}^2+g_{22}^2}{g_{21}^2-g_{22}^2}\frac{(g_{21}^2-g_{22}^2)[2(1+\mu)^2+3(1+2\mu)]+16m^4g_{21}(1+\mu)}{(g_{21}^2-g_{22}^2)(1+\mu)-4m^4g_{21}}\varepsilon$$
$$- \left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\left\{\frac{\mu m^2}{8(g_{21}^2-g_{22}^2)g_{21}}\left(\frac{g_{11}(g_{21}^2+g_{22}^2)+2g_{12}g_{21}g_{22}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{21}(g_{31}^2+g_{32}^2)(g_{21}^2+3g_{22}^2)+2g_{22}g_{31}g_{32}(3g_{21}^2+g_{22}^2)}{g_{21}^2-g_{22}^2}\right)\right.$$
$$+ \frac{\gamma_{24}m^2}{8(g_{21}^2-g_{22}^2)^2(1+\mu)}\left[\frac{(g_{21}^2+3g_{22}^2)(g_{31}^2-g_{32}^2)}{(1+\mu)} - (g_{21}^2+3g_{22}^2)(g_{31}^2+g_{32}^2) + 16g_{21}g_{22}g_{31}g_{32}\right.$$
$$\left.\left. - \frac{2g_{22}}{g_{21}^2(1+\mu)}(g_{21}g_{22}g_{32}(8g_{21}g_{32}+g_{22}g_{31}) - g_{21}^2g_{31}(g_{21}g_{32}+g_{22}g_{31}) + g_{22}^3g_{32}^2)\right]\right\}\varepsilon$$
$$+ \left(\frac{\gamma_{34}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^2}{8(g_{21}^2-g_{22}^2)g_{21}^2}\left\{\frac{2+\mu}{(1+\mu)^2}[g_{21}g_{31}(g_{21}^2+g_{22}^2)+g_{22}g_{32}(5g_{21}^2+g_{22}^2)] + [g_{22}g_{32}(2g_{21}^2-g_{22}^2)-g_{21}g_{31}(3g_{21}^2-2g_{22}^2)]\right\}\varepsilon\Bigg)$$

$$\lambda_p^{(4)} = \frac{1}{2}\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)^2 \frac{\gamma_{24}m^{10}(1+\mu)}{64(g_{21}^2-g_{22}^2)g_{21}}\left\{4\frac{g_{22}^2(3g_{21}^2+g_{22}^2)}{(g_{21}^2-g_{22}^2)^2} - 1 + \frac{(1+\mu)}{(g_{21}^2-g_{22}^2)(g_{23}^2-g_{24}^2)g_{21}}[g_{23}(g_{21}^2+g_{22}^2)(3g_{21}^2+g_{22}^2) - 2g_{24}g_{21}g_{22}(2g_{21}^2+g_{22}^2)] + \left((1+\mu)\frac{g_{21}^2+g_{22}^2}{g_{21}}\frac{g_{21}g_{23}-g_{22}g_{24}}{g_{23}^2-g_{24}^2} + 2g_{21}\frac{g_{21}g_{23}-g_{22}g_{24}}{g_{23}^2-g_{24}^2} + 3\frac{g_{21}^2+g_{22}^2}{g_{21}}\right)R_1 + \left((1+\mu)\frac{g_{21}^2+g_{22}^2}{g_{21}}\frac{g_{22}g_{23}-g_{21}g_{24}}{g_{23}^2-g_{24}^2} + 2g_{21}\frac{g_{22}g_{23}-g_{21}g_{24}}{g_{23}^2-g_{24}^2} - 2g_{22}\right)R_2\right\}\varepsilon^{-1} \tag{A2}$$

$$\lambda_s^{(0)} = -\frac{m}{2n\beta}\left\{\frac{\gamma_{24}m^2g_{22}}{(g_{21}^2-g_{22}^2)(1+\mu)}\varepsilon^{-1} + \gamma_{24}\frac{g_{21}g_{32}+g_{22}g_{31}}{g_{21}^2-g_{22}^2}\frac{(2+\mu)}{(1+\mu)^2} + \frac{1}{(1+\mu)m^2}\left[\frac{g_{12}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{22}(g_{31}^2+g_{32}^2)+2g_{21}g_{31}g_{32}}{g_{21}^2-g_{22}^2}\right]\varepsilon - \frac{\mu}{m^4(1+\mu)^2}\left[\frac{g_{11}g_{32}+g_{12}g_{31}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{22}g_{31}(g_{31}^2+3g_{32}^2)+g_{21}g_{32}(3g_{31}^2+g_{32}^2)}{g_{21}^2-g_{22}^2}\right]\varepsilon^2 + \frac{\mu^2}{m^6(1+\mu)^3}\left[\frac{g_{12}(g_{31}^2+g_{32}^2)+2g_{11}g_{31}g_{32}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{32}(g_{21}g_{31}+g_{22}g_{32})(3g_{31}^2+g_{32}^2)+g_{31}(g_{21}g_{32}+g_{22}g_{31})(g_{31}^2+3g_{32}^2)}{g_{21}^2-g_{22}^2}\right]\varepsilon^3\right\}$$

$$\lambda_s^{(2)} = -\frac{m}{2n\beta}\left\{\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^6g_{21}g_{22}}{2(g_{21}^2-g_{22}^2)^2}\varepsilon^{-1} - \left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^4g_{22}}{2(g_{21}^2-g_{22}^2)^2(1+\mu)}(g_{21}g_{31}-3g_{22}g_{32}) + \left(\frac{\gamma_{34}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^4g_{22}}{4(g_{21}^2-g_{22}^2)(1+\mu)}[(1+2\mu)+(1+\mu)^2] - \left(\frac{\gamma_{14}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^2}{8}\frac{g_{22}}{g_{21}}(1+2\mu)\varepsilon + \frac{1}{2}\frac{\gamma_{24}m^2n^4\beta^4g_{22}}{(g_{21}^2-g_{22}^2)(1+\mu)}\frac{(g_{21}^2-g_{22}^2)[2(1+\mu)^2+3(1+2\mu)]+16m^4g_{21}(1+\mu)}{(g_{21}^2-g_{22}^2)(1+\mu)-4m^4g_{21}}\varepsilon - \left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\left[\frac{\mu m^2}{8(g_{21}^2-g_{22}^2)g_{21}}\left(\frac{g_{12}(g_{21}^2+g_{22}^2)+2g_{11}g_{21}g_{22}}{\gamma_{14}} + \frac{\gamma_{24}}{(1+\mu)^2}\frac{g_{22}(g_{31}^2+g_{32}^2)(3g_{21}^2+g_{22}^2)+2g_{21}g_{31}g_{32}(g_{21}^2+3g_{22}^2)}{g_{21}^2-g_{22}^2}\right) + \frac{\gamma_{24}m^2}{8(g_{21}^2-g_{22}^2)^2g_{21}(1+\mu)}\left(\frac{g_{22}(3g_{21}^2+g_{22}^2)(g_{31}^2-g_{32}^2)}{1+\mu} - g_{22}(g_{31}^2+g_{32}^2)(3g_{21}^2+g_{22}^2) + 4g_{21}g_{31}g_{32}(g_{21}^2+3g_{22}^2) - \frac{2}{1+\mu}[g_{21}g_{22}g_{32}(4g_{21}g_{32}+g_{22}g_{31}) - g_{21}^2g_{31}(g_{21}g_{32}+g_{22}g_{31}) + 5g_{22}^3g_{32}^2]\right)\right]\varepsilon + \left(\frac{\gamma_{34}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{\gamma_{24}m^2}{8(g_{21}^2-g_{22}^2)g_{21}}\left(\frac{2(1+2\mu)}{(1+\mu)^2}[g_{32}(g_{21}^2+2g_{22}^2)+g_{21}g_{22}g_{31}] + [g_{32}(2g_{21}^2-g_{22}^2)-g_{21}g_{22}g_{31}]\right)\varepsilon\right\}$$

$$\lambda_s^{(4)} = -\frac{m}{2n\beta}\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)^2\frac{\gamma_{24}m^{10}(1+\mu)}{64(g_{21}^2-g_{22}^2)}\left\{4\frac{g_{22}(g_{21}^2+3g_{22}^2)}{(g_{21}^2-g_{22}^2)^2} - \frac{g_{22}}{g_{21}^2} + \frac{(1+\mu)}{(g_{21}^2-g_{22}^2)(g_{23}^2-g_{24}^2)g_{21}^2}[g_{24}(g_{21}^2-g_{22}^2)^2 + 2g_{21}g_{22}g_{23}(3g_{21}^2+g_{22}^2) - 2g_{21}^2g_{24}(g_{21}^2+3g_{22}^2)] + \left(\frac{(1+\mu)}{g_{21}^2}\frac{g_{22}g_{23}(3g_{21}^2-g_{22}^2)-g_{21}g_{24}(g_{21}^2+g_{22}^2)}{g_{23}^2-g_{24}^2} + 2\frac{g_{22}}{g_{21}}\frac{g_{21}g_{23}-g_{22}g_{24}}{g_{23}^2-g_{24}^2} + 6\frac{g_{22}}{g_{21}}\right)R_1 - \left(\frac{(1+\mu)}{g_{21}^2}\frac{g_{22}g_{24}(3g_{21}^2-g_{22}^2)-g_{21}g_{23}(g_{21}^2+g_{22}^2)}{g_{23}^2-g_{24}^2} - 2\frac{g_{22}}{g_{21}}\frac{g_{22}g_{23}-g_{21}g_{24}}{g_{23}^2-g_{24}^2} + \frac{g_{21}^2+g_{22}^2}{g_{21}^2}\right)R_2\right\}\varepsilon^{-1} \tag{A3}$$

$$\delta_x^{(2)} = \frac{1}{16}\left[\frac{b}{\pi\alpha}\frac{g_{22}^2}{g_{21}^2}\varepsilon^{1/2} + m^2(1+2\mu)\frac{g_{21}^2+g_{22}^2}{g_{21}^2}\varepsilon - 2\frac{g_{31}(g_{21}^2+g_{22}^2)-2g_{32}g_{21}g_{22}}{g_{21}^2}\varepsilon^2 + \frac{3}{m^2}\frac{(g_{21}^2+g_{22}^2)(g_{31}^2+g_{32}^2)-4g_{21}g_{22}g_{31}g_{32}}{g_{21}^2}\varepsilon^3 - \frac{2}{m^4}\frac{g_{31}(g_{21}^2+g_{22}^2)(g_{31}^2+3g_{32}^2)-2g_{32}g_{21}g_{22}(3g_{31}^2+g_{32}^2)}{g_{21}^2}\varepsilon^4\right]$$

$$\delta_x^{(4)} = \frac{1}{128}\left\{\frac{b}{32\pi\alpha}\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)^2\frac{m^8(1+\mu)^2}{n^4\beta^4g_{21}^2}\varepsilon^{-3/2} + 2\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{g_{21}^2+g_{22}^2}{g_{21}^3}(1+\mu)^2m^4(m^2-g_{31}\varepsilon)\varepsilon + m^2n^4\beta^4(1+\mu)^2\left(\frac{g_{21}^2-g_{22}^2}{g_{21}^2}\right)^2\left[\frac{(g_{21}^2-g_{22}^2)(1+2\mu)+8m^4g_{21}(1+\mu)}{(g_{21}^2-g_{22}^2)(1+\mu)-4m^4g_{21}}\right]^2\varepsilon^3\right\} \tag{A4}$$

In the preceding equations,

$$g_{11} = m^4 + 2\gamma_{12}m^2n^2\beta^2 + \gamma_{14}^2n^4\beta^4, \qquad g_{12} = 4mn\beta(\gamma_{11}m^2 + \gamma_{13}n^2\beta^2), \qquad g_{13} = m^4 + 18\gamma_{12}m^2n^2\beta^2 + 81\gamma_{14}^2n^4\beta^4$$

$$g_{14} = 12mn\beta(\gamma_{11}m^2 + 9\gamma_{13}n^2\beta^2), \qquad g_{21} = m^4 + 2\gamma_{22}m^2n^2\beta^2 + \gamma_{24}^2n^4\beta^4, \qquad g_{22} = 2mn\beta(\gamma_{21}m^2 + \gamma_{23}n^2\beta^2)$$

$$g_{23} = m^4 + 18\gamma_{22}m^2n^2\beta^2 + 81\gamma_{24}^2n^4\beta^4, \qquad g_{24} = 6mn\beta(\gamma_{21}m^2 + 9\gamma_{23}n^2\beta^2), \qquad g_{31} = \gamma_{30}m^4 + \gamma_{32}m^2n^2\beta^2 + \gamma_{34}n^4\beta^4$$

$$g_{32} = mn\beta(\gamma_{31}m^2 + \gamma_{33}n^2\beta^2), \qquad g_{33} = \gamma_{30}m^4 + 9\gamma_{32}m^2n^2\beta^2 + 81\gamma_{34}n^4\beta^4, \qquad g_{34} = 3mn\beta(\gamma_{31}m^2 + 9\gamma_{33}n^2\beta^2)$$

$$\Delta_1 = g_{21}(g_{23}^2 - g_{24}^2) - g_{23}(g_{21}^2 - g_{22}^2)(1+\mu)$$
$$\Delta_2 = 3g_{22}(g_{23}^2 - g_{24}^2) - g_{24}(g_{21}^2 - g_{22}^2)(1+\mu)$$
$$\Delta_3 = (g_{23}^2 - g_{24}^2) + (g_{21}g_{23} - g_{22}g_{24})(1+\mu)$$
$$\Delta_4 = (g_{22}g_{23} - g_{21}g_{24})(1+\mu)$$

$$R_1 = \frac{\Delta_3\Delta_1 + \Delta_4\Delta_2}{\Delta_1^2 - \Delta_2^2}, \qquad R_2 = \frac{\Delta_3\Delta_2 + \Delta_4\Delta_1}{\Delta_1^2 - \Delta_2^2}$$
$$C_3 = 1 - \frac{g_{21}g_{31} - g_{22}g_{32}}{m^2 g_{21}}\varepsilon$$

$$b = \left(\frac{\gamma_{14}\gamma_{24}}{1 + \gamma_{14}\gamma_{24}\gamma_{30}^2}\right)^{1/2}, \qquad c = -\frac{\gamma_{14}\gamma_{24}\gamma_{30}}{1 + \gamma_{14}\gamma_{24}\gamma_{30}^2}$$
$$\alpha = \left[\frac{b-c}{2}\right]^{1/2}, \qquad \phi = \left[\frac{b+c}{2}\right]^{1/2}$$
$$C_{11} = \frac{C_{pt}}{g_t}, \quad C_{pt} = \gamma_{24}^2 - \frac{4\alpha}{\pi b}\gamma_5^2\varepsilon^{1/2}, \quad C_{st} = \gamma_{23} + \frac{2\alpha}{\pi b}\gamma_5\gamma_{21}\varepsilon^{1/2}$$

$$g_t = (\gamma_{24}^2\gamma_{T1} - \gamma_5\gamma_{T2} + \gamma_{23}\gamma_{T3}) + \frac{4\alpha}{\pi b}\gamma_5(\gamma_{T2} - \gamma_5\gamma_{T1} + \gamma_{21}\gamma_{T3})\varepsilon^{1/2} \tag{A5}$$

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